Solutions:

1. **Sun**

a) Understanding the Black Body radiation

The only difficulty in this problem is to find the density of states $D_\gamma(\hbar\omega)$. Since photons are quantum mechanical particle the obey Heisenberg's uncertainty relation*:

$$\Delta x \Delta p \geq h$$

In an object of finite volume V a photon is totally delocalized and one gets a relation for Δp^3:

$$\Delta p^3 = \frac{h^3}{V}.$$

For photons hold the following relation between energy and momentum:

$$pc = \hbar\omega$$

and so the number of states is contained inside a sphere with radius p:

$$N_\gamma = \frac{4}{3}\pi\frac{p^3}{\Delta p^3} = \frac{4}{3}\pi\frac{V(\hbar\omega)^3}{\mathrm{h}^3 c^3}$$

An additional factor of two for the number of states comes from the two possibilities of polarization. The density of states follows from the total number of states by differentiating:

$$D_\gamma(\hbar\omega) = 2\frac{\partial N_\gamma}{\partial V \partial(\hbar\omega)} = \frac{(\hbar\omega)^2}{\pi^2\hbar^3 c^3}$$

The distribution function for photons is the Bose-Einstein distribution function:

$$f_\gamma(\hbar\omega) = \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T} - \mu_\gamma\right) - 1}$$

The chemical potential for photons is zero. As a "simple" interpretation of that fact one can say, that no energy is needed for the process of producing a photon. From this the following expression follows:

$$\frac{dn_\gamma(\hbar\omega)}{d\hbar\omega} = \frac{D_\gamma(\hbar\omega)}{4\pi} f_\gamma(\hbar\omega) d\Omega = \frac{(\hbar\omega)^2}{4\pi^3\hbar^3 c^3}\frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1} d\Omega$$

For the energy density it follows then:

$$\frac{de_\gamma(\hbar\omega)}{d\hbar\omega} = \hbar\omega\,\frac{dn_\gamma(\hbar\omega)}{d\hbar\omega}$$

$$\Rightarrow e_\gamma = \frac{\pi^2 k^4}{15\hbar^3 c^3} T^4$$

** This restricts the number of photons in a certain phase space volume. This is quite important since for classical point-like objects it holds that $\Delta x \Delta p = 0$. This would make it possible to have an infinite number of photons in a cavity, which however would lead to an infinite energy density resulting the famous ultraviolet catastrophe.*

b) Radiation from the sun towards the earth

We assume that the sun is a perfect black body. The radiation power can be obtained by calculating the energy current density and then the energy current (compare the units). An expression for the energy current density is similar to a charge current density**:

$$dj_{Sun} = \frac{e_\gamma}{4\pi} \cdot c\, d\Omega$$

To obtain the radiation power it is then needed to integrate over the whole area of the black body:

$$dP_{Sun} = dj_{Sun} dA_{Sun}$$
$$P_{Sun} = \frac{e_\gamma}{4} c\, \pi r_{Sun}^2$$
$$= \sigma T^4 A_{Sun} \approx 3.85 \cdot 10^{26} W$$

By the calculation above one has to be careful with the definition of the solid angle with respect to the surface element dA: $d\Omega = dA/R^2 = 2\pi \cos\theta \sin\theta\, d\theta$.

The amount of energy per day which the earth receives can be calculated by adapting the solid angle and integrating over the time.

$$dE_{Earth} = \frac{e_\gamma}{4\pi} \cdot c\, d\Omega_{\text{Earth}} dA_{sun} dt$$
$$E_{Earth} = \pi r_{Earth}^2 \underbrace{\frac{P_{Sun}}{4\pi R_{ES}^2}}_{S=1361\frac{W}{m^2}} t_{Day} \approx 1.5 \cdot 10^{22} J$$

The air mass is affecting the results by a different e_γ! The air mass is determined by the absorption of different gases in the atmosphere.

*** The correspondence to an electronic density current follows by identifying $e_\gamma = \rho_C$ and $c = v$.*

c) Modeling a real solar cell, Part 1

In the lecture the following equation was obtained:

$$S_{module} = S_{horizontal} \frac{\sin(\alpha + \beta + \delta)}{\sin(\alpha + \delta)}$$

On the 21st of March $\delta = 0°$ and Karlsruhe is at $\alpha = 49°$.

2. Carrier dynamics in Semiconductors

a) Diffusion current density

With the starting point for the current density:

$$j = q\phi = q\frac{\bar{l}}{2\tau}\big(n(x) - n(x+l)\big)$$

one can find the following expression by a Taylor expansion for small l:

$$\begin{aligned} j_D &\approx q\frac{\bar{l}}{2\tau}\left(n(x) - n(x) - l\frac{\partial n(x)}{\partial x}\right) \\ &= -q\frac{l\bar{l}}{2\tau}\frac{\partial n(x)}{\partial x} = -qD\frac{\partial n(x)}{\partial x} \end{aligned}$$

b) Field induced current density

For this problem one needs Ohm's law:

$$j = \sigma E$$

and the relation between the conductivity σ and the mobility μ:

$$\sigma = qn(x)\mu$$

Inserting this into Ohm's law leads to:

$$j_F = \sigma E = q\mu n(x)E$$

c) Diffusion equation

Continuity equation:

$$\partial_t \rho + \partial_x j = 0$$

Inserting both expressions for the current and making the assumption that the external field is homogenous leads to:

$$\partial_x(j_F + j_D) = -qD\frac{\partial^2 n(x)}{\partial x^2} + q\mu E\frac{\partial n(x)}{\partial x}$$

Recombination and generation of charge carriers:

$$U - G$$

3. **Semiconductor**

a) **What is a semiconductor?**

Since electrons are fermions there distribution is the Fermi-Dirac distribution function:

$$f(E) = \frac{1}{\exp\left(\frac{E - E_F}{kT}\right) + 1}$$

The characteristic of this function is that for $E < E_F, f(E) \approx 1$ and for $E > E_F, f(E) \approx 0$. To obtain the behavior at $E = E_F$ at $T = 0K$ one has to differentiate the function with respect to E.

$$f'(E)|_{T=0} = \infty$$

So at $T = 0\ K$ the distribution function is a step function.

b) **Everything is written in the slides**

c) **Diffusion potential**

In the equilibrium state there is no current flow so:

$$\frac{\partial n}{\partial t} = 0 \Rightarrow j_F = -j_D$$

$$e\mu n(x)E(x) = -eD\partial_x n(x)$$

$$\frac{\mu}{D}\frac{d\phi_{el}(x)}{dx} = -\frac{1}{n}\frac{dn(x)}{dx}$$

$$\frac{e}{kT}\int_{\phi_p} d\phi = \frac{e}{kT}U_D = \ln\left(n_n/n_p\right)$$